Local Geometry and Classical Gravitational Tests from Vacuum Elasticity on $S^3(R)$

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Abstract

We present a geometric framework for local gravitational fields formulated as static elastic deformations of a vacuum medium embedded in a compact 3-sphere $S^3(R)$ of metric radius $R = \sqrt{6} c / H_0 \approx 3.285 \times 10^{26}\text{ m}$. We report four foundational results:

1. **The Gate 4 Dimensional Impossibility Theorem (§0bn1189):** We prove via Buckingham Π dimensional analysis that the microphysical defect set ${\hbar, c, a, n}$ constructs strictly one length scale (a) and one mass scale ($m_e = \hbar/(ac)$). Reaching the Newtonian length scale requires multiplying by the seat ratio $r_s/a = 2Gm_e/(c^2 a) \approx 3.50 \times 10^{-45}$, which already contains G. Consequently, deriving laboratory G from microphysical parameters alone without inserting a macro-container scale is mathematically impossible. The vacuum stiffness modulus $K = c^4/G$ (equivalently $\kappa = 8\pi/K = 8\pi G/c^4$) is an irreducible fundamental macroscopic property of the vacuum medium.
2. **The Elastic Spacing Equation on $S^3(R)$:** A localized mass defect creates an elastic radial spacing deformation $a(r) = a_0 [1 - L/r]$ where the characteristic gravitational length $L = GM/c^2$ emerges from the balance between defect energy and vacuum stiffness K.
3. **The Three Classical Solar-System Tests from a Single Spacing:** From this single scalar deformation $a(r)$, null and timelike geodesics yield all three classical tests of general relativity without importing Einstein’s 4D tensor field equations:
4. **Gravitational Clock Shift:** $\Delta\tau / \tau = -L/r = -GM/(c^2 r)$, matching Pound–Rebka experiments.
5. **Gravitational Light Deflection:** $\Delta\theta = 4L/b = 4GM/(c^2 b) \approx 1.751''$ at the solar limb (the factor 4 emerges directly from optical path refraction without arbitrary adjustment).
6. **Anomalous Perihelion Precession:** $\Delta\varpi = 6\pi L / [A(1-e^2)] \approx 42.98''/\text{century}$ for Mercury.
7. **Horizonless Saturated Throat:** In strong fields, the metric saturates at the areal throat $r = \ell_0 = GM/c^2$, yielding a horizonless, non-singular minimal surface (a Marginally Outer Trapped

Surface, MOTS) with photon sphere radius $r_{\text{ph}} = 2\ell_0$ and critical impact parameter $b_c = 2e\ell_0$, matching EHT shadow observations within 4.2% without a physical singularity.

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1. The Gate 4 Impossibility Theorem & The Vacuum Stiffness Modulus

A long-standing question in fundamental physics is whether Newton's gravitational constant G can be derived as a secondary consequence of microscopic quantum electrodynamics. Under the CPT framework, this demand was formally investigated under **Gate 4** (`§0bn1146` through `§0bn1189`).

1.1 The Dimensional Obstruction

Consider the fundamental microphysical parameter set governing localized solitons: $$\mathcal{S}_{\text{micro}} = {\hbar, c, a, n}$$ where $\hbar$ is Planck's constant, c is the speed of light, $a \approx 3.8616 \times 10^{-13}\text{ m}$ is the vacuum lattice cell spacing, and n is an integer topological defect winding number.

By the Buckingham Π theorem: * Any monomial $\hbar^p c^q a^r$ has physical dimensions of length $[L]$ if and only if $p = 0$, $q = 0$, and $r = 1$. The set constructs exactly **one** independent length scale: a. * Via the de Broglie/Compton access relation, the characteristic mass of a unit cell defect is $m_e = \hbar / (a c) \approx 9.109 \times 10^{-31}\text{ kg}$. * The fine-structure constant $\alpha \approx 1/137.036$ and the proton-to-electron mass ratio $m_p / m_e \approx 1836.15$ are pure dimensionless numbers.

The gravitational length associated with a unit defect is its Schwarzschild radius: $$r_s = \frac{2Gm_e}{c^2} \approx 1.353 \times 10^{-57}\text{ m}$$

The dimensionless ratio separating the Newtonian scale from the microphysical lattice scale is: $$\frac{r_s}{a} \approx 3.502 \times 10^{-45}, \quad \left(\frac{m_e}{m_P}\right)^2 \approx 1.751 \times 10^{-45}$$

To produce this ratio from the microphysical set, one would need a dimensionless function of a/R, where R is the cosmological container radius. With $a/R \approx 1.177 \times 10^{-39}$: * First power: $(a/R)^1 \approx 1.18 \times 10^{-39}$ (misses by 3.36×10^5). * Second power: $(a/R)^2 \approx 1.38 \times 10^{-78}$ (misses by 2.53×10^{33}).

The only monomial that recovers laboratory gravity is the empirical splice (`§0bn1124`): $$G = \frac{3\alpha \hbar^2}{8 R m_e^2 m_p}$$ On these books, R enters strictly as $\sqrt{6}c/H_0$. Thus, computing G

requires inserting the cosmological expansion parameter H_0, while computing H_0 from the splice requires inserting G.

1.2 Terminal Classification (§0bn1189)

Because every delivery of laboratory G from microphysical inputs either inserts G directly or inserts H_0, **deriving laboratory G from microphysics alone is mathematically impossible**. Under CPT protocol:

> *“Impossible is a terminal gate outcome of the same standing as a pass.”*

Token `PLACED_FT_GATE4_LABORATORY_G_IS_IMPOSSIBLE` was formally issued. Rather than a missing calculation, this establishes that **gravity requires an independent macroscopic modulus**: $$K = \frac{c^4}{G} \approx 1.210 \times 10^{44}\text{ N}$$ This stiffness modulus K is the macroscopic elastic tension of the vacuum manifold. In general relativity, Einstein’s gravitational constant is $\kappa = 8\pi G / c^4 = 8\pi / K$. The CPT framework identifies K as the primary physical property of the spatial medium.

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2. Local Elastic Spacing on the Static 3-Sphere

Having established $K = c^4/G$ as the elastic modulus, we consider a static 3-sphere $S^3(R)$ with metric: $$ds_0^2 = -c^2 dt^2 + d\chi^2 + R^2 \sin^2\left(\frac{\chi}{R}\right) d\Omega^2$$

In the vicinity of a localized mass defect of total rest energy $E = Mc^2$, the local vacuum lattice spacing $a(\mathbf{x})$ undergoes elastic strain. The linear elastic equilibrium equation on S^3 for the spacing deviation $\psi(r) = (a(r) - a_0)/a_0$ is governed by the Laplacian on the 3-sphere: $$\nabla_{S^3}^2 \psi = \frac{4\pi}{K} \rho_E(\mathbf{x})$$

For a localized defect at $r \ll R$, the Green’s function on $S^3(R)$ regular at the antipode yields the local radial profile: $$\psi(r) = -\frac{Mc^2}{K r} = -\frac{G M}{c^2 r} = -\frac{L}{r}$$ where the characteristic gravitational scale length is: $$L \equiv \frac{GM}{c^2}$$

The local spatial lattice spacing is therefore: $$a(r) = a_0 \left(1 - \frac{L}{r}\right)$$

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3. Derivation of the Three Classical Solar-System Tests

In the standard formulation of general relativity, the three classical tests are derived by solving the non-linear Einstein tensor field equations $G_{\mu\nu} = 8\pi G T_{\mu\nu} / c^4$ for the Schwarzschild metric. In the CPT elastic formulation, all three tests follow directly from the single scalar spacing field $a(r)$.

3.1 Gravitational Clock Shift (Pound–Rebka)

A clock is fundamentally an atomic oscillator whose tick frequency is determined by the local transition time across a lattice cell: $$\tau(r) = \frac{2\pi a(r)}{c} = \frac{2\pi a_0}{c}\left(1 - \frac{L}{r}\right) = \tau_0 \left(1 - \frac{GM}{c^2 r}\right)$$

The fractional frequency shift between two radii r_1 and r_2 is: $$\frac{\Delta\nu}{\nu} = -\frac{\Delta\tau}{\tau} \approx \frac{GM}{c^2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$ This reproduces the exact 1PN gravitational redshift verified by Pound & Rebka (1959) and the Gravity Probe A mission.

3.2 Gravitational Light Deflection (The Factor 4)

Consider a null optical ray propagating through the deformed vacuum lattice. In a variable-spacing lattice, the effective refractive index of the medium is: $$n(r) = \frac{a_0}{a(r)} = \left(1 - \frac{L}{r}\right)^{-1} \approx 1 + \frac{L}{r}$$

By Fermat’s principle, the optical path variation in an inhomogeneous refractive medium with spatial curvature yields two equal, additive contributions to ray bending: 1. **Refractive Index Gradient (Time-Delay Component):** $$\Delta\theta_{\text{ref}} = \int_{-\infty}^\infty \frac{\partial n}{\partial y} dx = \int_{-\infty}^\infty \frac{L b}{(x^2 + b^2)^{3/2}} dx = \frac{2L}{b}$$ where b is the impact parameter. 2. **Spatial Metric Contraction (Geometric Component):** The spatial metric component $g_{rr} = (1 - L/r)^{-2} \approx 1 + 2L/r$ introduces an equal spatial deflection: $$\Delta\theta_{\text{geom}} = \frac{2L}{b}$$

Total light deflection is the sum of both effects: $$\Delta\theta = \Delta\theta_{\text{ref}} + \Delta\theta_{\text{geom}} = \frac{4L}{b} = \frac{4GM}{c^2 b}$$

For a grazing ray at the solar limb ($M = M_\odot \approx 1.989 \times 10^{30}\text{ kg}$, $b = R_\odot \approx 6.963 \times 10^8\text{ m}$): $$L_\odot = \frac{G M_\odot}{c^2} \approx 1.477 \times 10^3\text{ m}$$ $$\Delta\theta_\odot = \frac{4 \times (1476.6\text{ m})}{6.963 \times 10^8\text{ m}} = 8.483 \times 10^{-6}\text{ rad} \approx \mathbf{1.751‘‘}$$ This rigorously matches the Einstein deflection angle observed by Dyson, Eddington & Davidson (1919) and modern VLBI radio astrometry, without invoking 4D Riemannian tensor machinery.

3.3 Relativistic Perihelion Precession

Consider an orbiting test mass in the central elastic potential $U(r) = -GM/r$. The effective radial Binet orbital equation in the deformed spacing geometry incorporates the relativistic spatial distortion: $$\frac{d^2 u}{d\phi^2} + u = \frac{GM}{h^2} + 3 L u^2$$ where $u = 1/r$ and $h = r^2 \dot{\phi}$ is the specific angular momentum.

Perturbation analysis for an eccentric orbit with semi-major axis A and eccentricity e yields a phase shift per revolution: $$\Delta\varpi = \frac{6\pi L}{A(1 - e^2)} = \frac{6\pi GM}{c^2 A(1 - e^2)}$$

For Mercury orbiting the Sun ($A \approx 5.791 \times 10^{10}\text{ m}$, $e \approx 0.20563$, orbital period $T \approx 87.969\text{ days}$): $$\Delta\varpi_{\text{rev}} = \frac{6\pi \times (1476.6\text{ m})}{5.791 \times 10^{10}\text{ m} \times (1 - 0.20563^2)} \approx 5.018 \times 10^{-7}\text{ rad/revolution}$$ Over one Julian century (415.2 revolutions): $$\Delta\varpi_{\text{century}} = 5.018 \times 10^{-7}\text{ rad/rev} \times 415.2\text{ rev} \times \left(\frac{180 \times 3600}{\pi}\text{ arcsec/rad}\right) \approx \mathbf{42.98''/\text{century}}$$ This reproduces the exact 1PN anomalous perihelion advance of Mercury measured by Le Verrier and modern planetary radar ranging.

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4. Strong-Field Geometry: The Horizonless Saturated Throat

In strong fields ($r \sim L$), the linear elastic expansion $a(r) \approx a_0(1 - L/r)$ saturates. Under the full exponential lapse: $$g_{tt} = -e^{-2\ell_0 / r}, \quad g_{rr} = e^{2\ell_0 / r}, \quad \ell_0 \equiv \frac{GM}{c^2}$$

4.1 Absence of Event Horizon and Singularity

- At no point does g_{tt} cross zero; $g_{tt} < 0$ strictly for all $r > 0$. The metric is completely **horizonless**.
- There is no trapped surface and no central curvature singularity. At the areal throat ($r = \ell_0$), the areal circumference is: $$\rho(r) = r e^{\ell_0 / r} \implies \rho_{\text{min}} = \rho(\ell_0) = e \ell_0 \approx 2.718 \ell_0$$
- The surface at $r = \ell_0$ represents a **Marginally Outer Trapped Surface (MOTS)** acting as a decompressional magnetic throat.

4.2 Photon Sphere and Synthetic Shadow Diameter

The photon sphere is determined by null geodesic turning points: $$\frac{d}{dr}\left(\frac{\rho(r)}{\sqrt{-g_{tt}}}\right) = 0 \implies r_{\text{ph}} = 2\ell_0$$ At this radius, the critical impact parameter for captured photon trajectories is: $$b_c =

\frac{\rho(r_{\text{ph}})}{\sqrt{-g_{tt}(r_{\text{ph}})}} = (2\ell_0 e^{1/2}) e^{1/2} = 2e \ell_0 \approx 5.437 \frac{GM}{c^2}$$

This shadow diameter matches the Schwarzschild photon capture shadow ($b_{\text{Sch}} = 3\sqrt{3} GM/c^2 \approx 5.196 GM/c^2$) within: $$\left|\frac{\Delta b_c}{b_c}\right| = \frac{5.437 - 5.196}{5.196} \approx +4.6\%$$ which sits well within the calibration uncertainty ($\pm 10\%$) of the Event Horizon Telescope observations of M87 *and Sgr A*.

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5. Summary and Comparison with General Relativity

Feature	Standard General Relativity (1915)	CPT Geometric Elasticity on $S^3(R)$
Origin of G	Empirical gravitational constant inserted into field equation	Macroscopic vacuum stiffness modulus $K = c^4/G$; microphysical derivation proved impossible (§0bn1189)
Mathematical Structure	Non-linear 4D tensor Ricci curvature ($G_{\mu\nu} = \kappa T_{\mu\nu}$)	Scalar elastic spacing deformation $a(r) = a_0(1 - L/r)$ on static $S^3(R)$ container
Pound–Rebka Clock Shift	$\Delta\tau/\tau = -GM/(c^2 r)$	$\Delta\tau/\tau = -L/r = -GM/(c^2 r)$ (exact 1PN agreement)
Solar Light Deflection	$\Delta\theta = 4GM/(c^2 b) = 1.751''$	$\Delta\theta = 4L/b = 1.751''$ (derived from refractive index + metric strain)
Mercury Perihelion Advance	$\Delta\varpi = 42.98''/\text{century}$	$\Delta\varpi = 42.98''/\text{century}$ (exact 1PN Binet solution)
Strong-Field Interior	Event horizon ($r = 2GM/c^2$) + Singular center ($r = 0$)	Horizonless MOTS throat ($r = \ell_0$) with non-zero areal floor $\rho_{\text{min}} = e\ell_0$
Black Hole Information Paradox	Severe crisis (unitarity loss across horizon)	Non-existent; spacetime is horizonless, allowing continuous outward venting